

On the Unification of Fundamental Particle Interactions

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Introduction.

- The search for a Unified Theory of Interactions has attracted the imagination of physicists for centuries
- There has been important progress in this direction, but there is still much to be learned
- Known fundamental particle interactions vary in their range, strength and nature
- Three of the four forces, however, share a common property: They are associated with quantum field theories, governed by gauge symmetries.
- We believe all forces should have a common origin.
Is there evidence that this is indeed true ?

Unifying forces in Physics

- The first gigantic step was performed by Newton, by providing a description of gravity consistent with the laws of classical mechanics
- The Maxwell's Equations present for the first time a Unified description of two dissimilar forces in the form of a single Gauge Theory
- They Unify the description of Electric and Magnetic Interactions
- In the absence of charge Sources, Maxwell's Equations

$$\begin{aligned}\vec{\nabla} \vec{E} &= \rho, & \vec{\nabla} \times \vec{B} - \frac{\partial \vec{E}}{\partial t} &= \vec{j} \\ \vec{\nabla} \vec{B} &= 0, & \vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} &= 0\end{aligned}\tag{1}$$

possess a dual description, a symmetry $\vec{E} \rightarrow \vec{B}, \vec{B} \rightarrow -\vec{E}$ ($F^{\mu\nu} \rightarrow \tilde{F}^{\mu\nu}$)

- Dual description broken by sources, due to the absence of magnetic monopoles

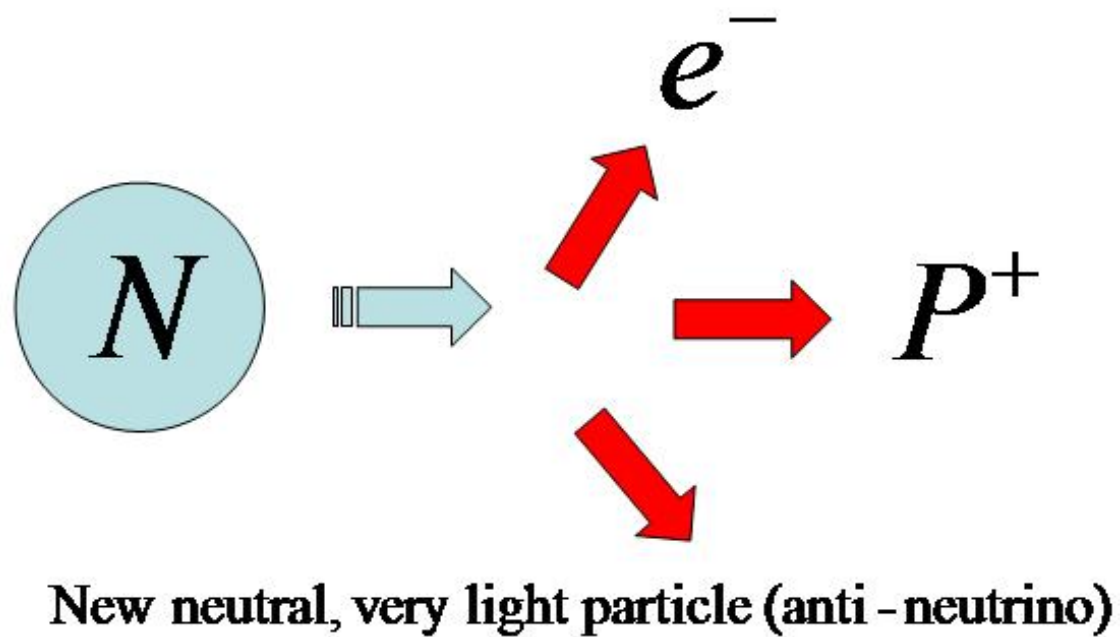
Towards Modern Unification

- Einstein (and Poincare) made the description of Electromagnetic interactions consistent with the laws of mechanics
- Special Relativity Emerged. Invariance under Lorentz Transformations.
- Heisenberg and Schroedinger define the rules of quantum mechanics
- Einstein's General Relativity replaces Newton's formulation of gravity
- Dirac marries quantum mechanics with special relativity: Relativistic Quantum Mechanics
- Feynman, Schwinger and Tomonaga make crucial contributions towards the development of Quantum Electrodynamics
- QED : Successful theory that describes the interactions of quanta of Electromagnetic Fields (Photons) with Charged Particles

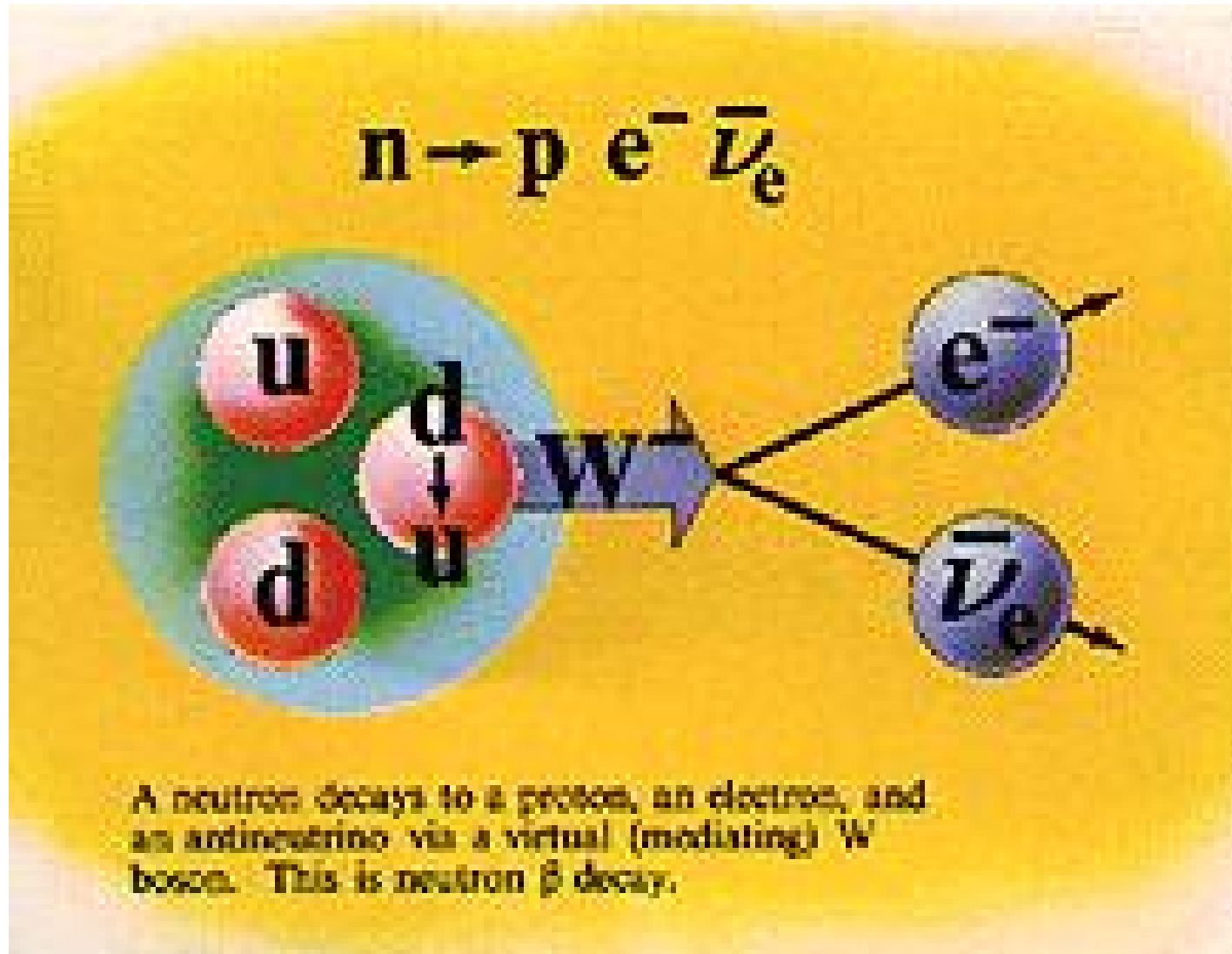
States of Confusion and Enlightening

- Beta decay of neutrons into a proton and an electron is discovered. New, neutral, light particle postulated by Pauli: The neutrino.
- Pions are identified as mediators of forces between nucleons
- After many years of confusion, a clear picture emerged
- Nucleons are formed by partons (quarks), glued together by strong forces, mediated by gluons, vector bosons similar to the photon.
- Beta decay is a manifestation of the weak interactions, mediated by heavy charged Vector bosons, with masses of order 100 GeV.
- All observable phenomena seems to be described by gravity, electromagnetism, plus the weak and strong interactions
- Progress towards the elucidation of the nature of these interactions demanded high energy experiments and radical ideas

Beta decay of a Neutron



Modern Understanding of Beta Decay



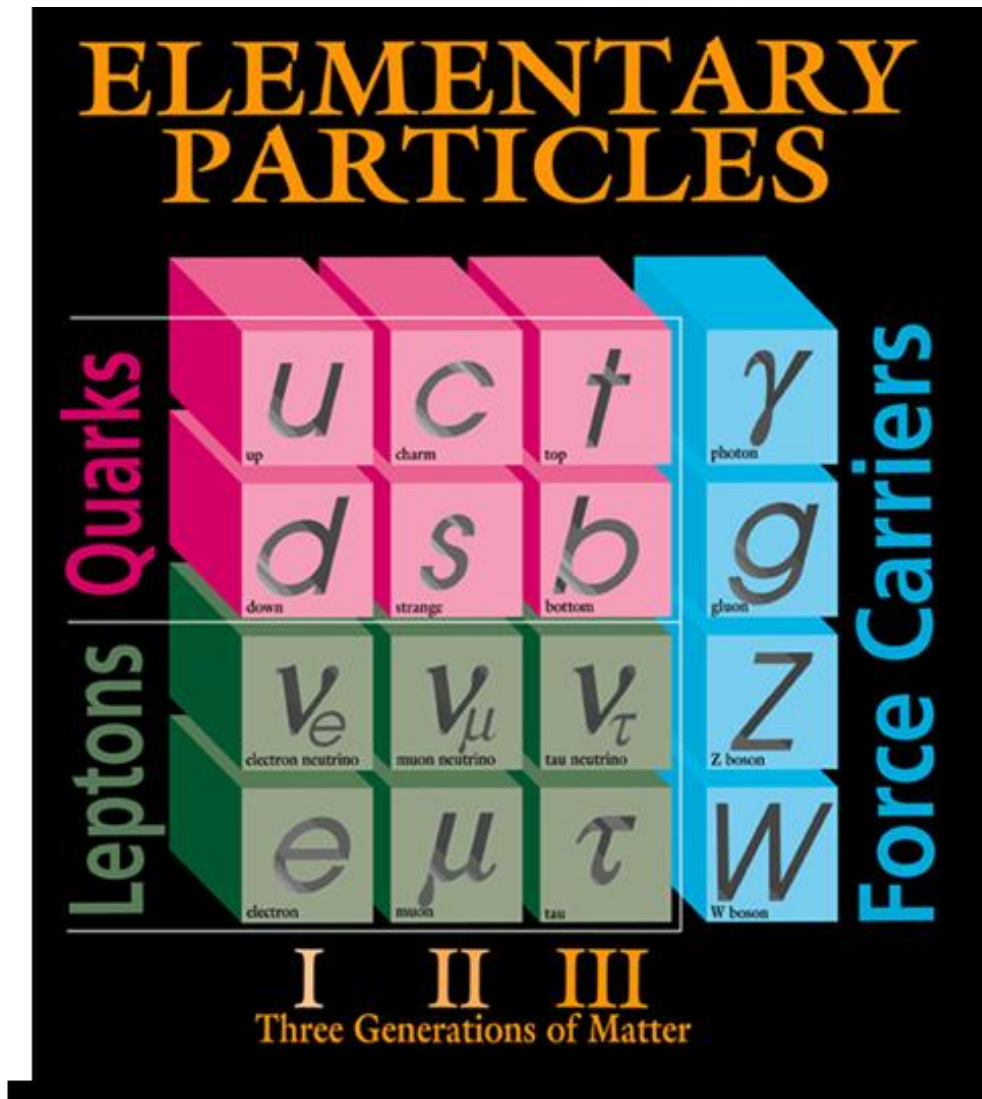
Complexity

Four forces of nature quite different

- Gravity: Long Range. Extremely weak between elementary particles. Mediated by massless gravitons. Affects all particles.
- Electromagnetic: Long range. Moderately strong. Affects only charged particles
- Strong: Short range. Forces become stronger at larger distances. No isolated charges. Confinement. Affects only quarks and gluons.
- Weak: Very short range. Forces mediated by massive charged and neutral vector bosons, W^{\pm} , Z^0 . Affects quarks and leptons.

How can these forces admit a common description ?

Standard Model particle content



Quantum Numbers of SM particles

$$\text{SM particle} \quad G_{SM} \equiv SU(3)_c \times SU(2)_L \times U(1)_Y$$

(S = 1/2) (3 generations)

$$Q = (t, b)_L \quad (3, 2, 1/6)$$

$$L = (\nu, l)_L \quad (1, 2, -1/2)$$

$$U = t_R \quad (3, 1, -2/3)$$

$$D = b_R \quad (3, 1, 1/3)$$

$$E = l_R \quad (1, 1, 1)$$

(S = 1)

$$B_\mu \quad (1, 1, 0)$$

$$W_\mu \quad (1, 3, 0)$$

$$g_\mu \quad (8, 1, 0)$$

Apart from quark, leptons and gauge bosons, a scalar Higgs doublet is necessary for the generation of mass

(S = 0)

$$H \quad (1, 2, 1/2)$$

Unified description of Electroweak Interactions

- Electromagnetic, Weak and Strong Forces described by gauge theories
- Charged and Neutral Massive Vector Bosons, mediating weak interactions have different charges and masses. Their couplings to quarks and leptons are also quite different.
- Charged Vector Bosons couple only to left-handed chiral fermions. Neutral Vector Bosons couple to both left- and right-handed quarks and leptons
- Weak and Electromagnetic Interactions proceed from a common gauge theory, based on the gauge group

$$SU(2)_L \times U(1)_Y.$$

One gauge boson associated with each generator. Three couple with coupling strength g_2 and one with coupling g_1 .

- Up and down quarks, and electrons and neutrinos form fundamental representations of $SU(2)_L$. Gauge bosons $W^{1,2}$ leads to the charged W^\pm . What about the neutral components?

Interaction of Neutral Currents

- Electromagnetic and Weak Interactions proceed from the breakdown of

$$SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$$

Glashow, Weinberg, Salam

- Breakdown induced by vacuum condensate of an $SU(2)_L$ doublet scalar field, $(0, v)$, with non-trivial charges under $U(1)_Y$. (Similar to superconductivity).
- $U(1)_{em}$ symmetry, governed by combination of diagonal generators remains unbroken.
- Mass eigenstates:

$$\begin{aligned}\gamma_\mu &= \cos \theta_W B_\mu + \sin \theta_W W_\mu^3 \\ Z_\mu &= -\sin \theta_W B_\mu + \cos \theta_W W_\mu^3\end{aligned}\tag{2}$$

with $\sin \theta_W = g_1/(g_1^2 + g_2^2)^{1/2}$ and

$$\begin{aligned}m_\gamma &= 0 \quad , \quad m_Z^2 = \frac{(g_2^2 + g_1^2)}{2} v^2 \\ e &= \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}} \quad , \quad Q = T_3 + Y\end{aligned}\tag{3}$$

Hints of a common structure

- The gauge symmetries that govern the SM are subject to anomalies
- An anomalous symmetry is one that, although present at the classical level, it is broken at the quantum level
- This breakdown is connected with the process of renormalization: It is impossible to regularize the divergences without breaking some global or local symmetries
- The preservation of the chiral $SU(2)_L$ symmetry requires a magic relationship between the charges of quarks and leptons:

$$\sum_{i=q,l} Q_i = 0 \tag{4}$$

- Such magic relationship is verified because of the presence of three quark colours, and can only be explained in a natural way if quarks and leptons form common multiplets in a larger symmetry gauge group

Running Couplings

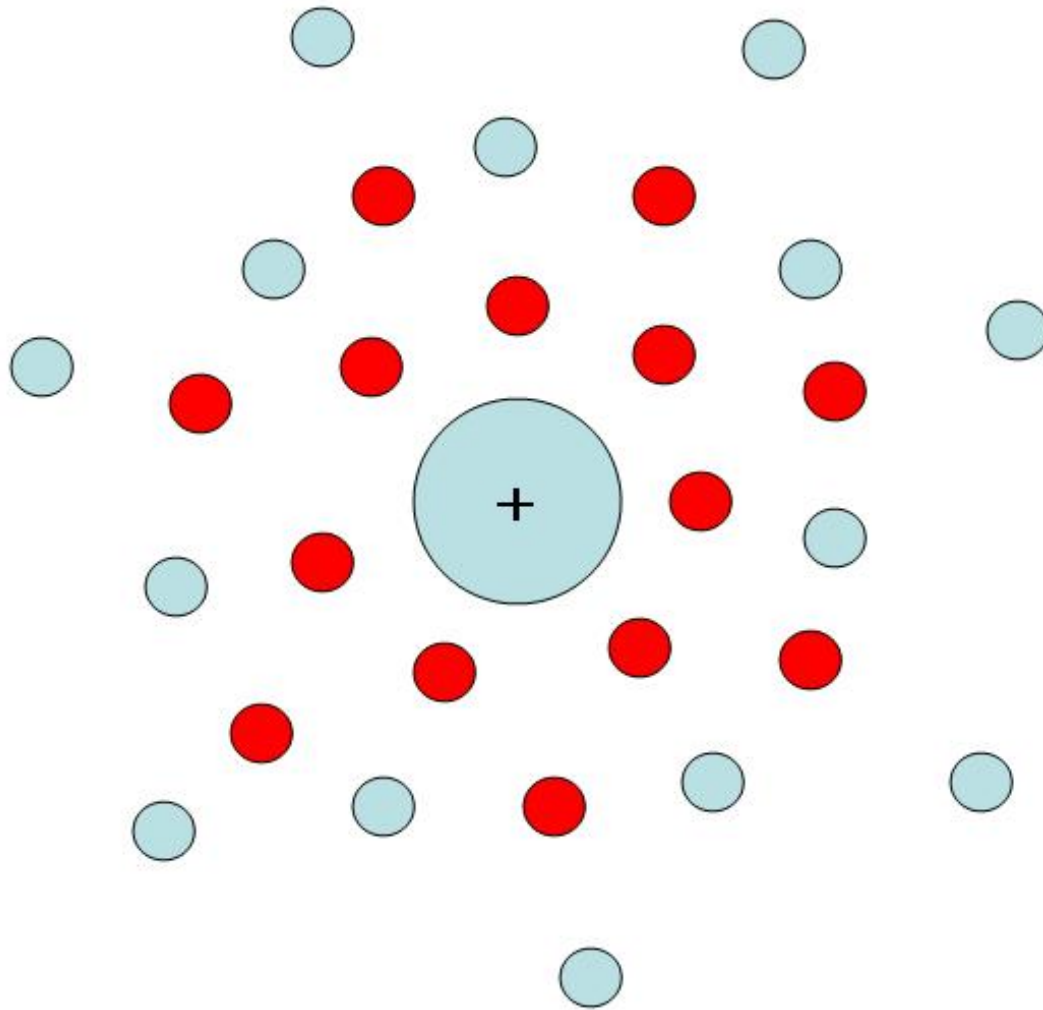
- In quantum field theory the vacuum is not empty.
- In the Dirac picture, for instance, it consists of a Dirac sea of negatively charged particles
- A charged particle polarizes the vacuum, leading to a screening of the electromagnetic charge at large distances
- The effective charge at a certain scale $Q = 1/r$ is governed by RG equations

$$\frac{d\alpha}{d\ln Q} = -\frac{b}{2\pi}\alpha^2 \quad (5)$$

$$\alpha(Q) = \frac{\alpha(M)}{1 + \frac{b\alpha(M)}{2\pi} \ln\left(\frac{Q}{M}\right)}, \quad \alpha(M) = \frac{\alpha(Q)}{1 - \frac{b\alpha(Q)}{2\pi} \ln\left(\frac{Q}{M}\right)} \quad (6)$$

- In QED, $b < 0$ and hence we find that, for a given value of α measured at a given low scale M , $\alpha(Q)$ gets strong at large Q 's (Landau Pole). In QED, however, it gets strong at scales larger than M_{Pl} .

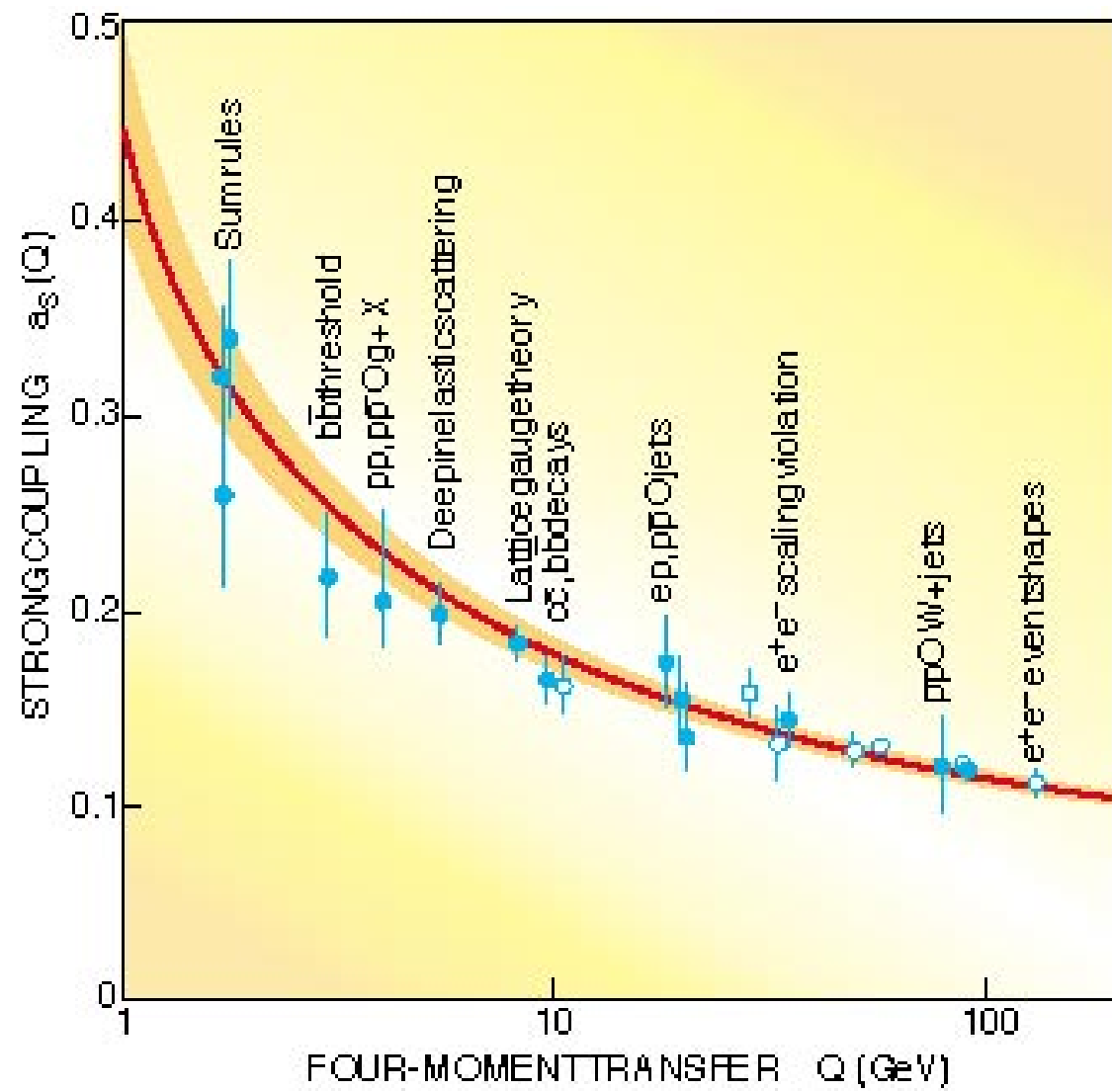
Screening of positive Electromagnetic Charge



A positive charge (blue) is located in the vacuum and it is screened by negative charges (red) arising from vacuum polarization effects. Only at very short distances (large Q) do we see the fundamental charge.

Strong Interactions. Asymptotic Freedom

- It was not until 1973 that, after the works of Gross, Wilczek and Politzer, with some insight from t'Hooft and Coleman, it was realized that, in non-abelian theories, the beta function coefficient b could be positive.
- Positive values of b imply a theory that becomes strongly coupled at long distances and weakly coupled at short distances
- When $b > 0$, particles are bounded by an increasingly strong force at long distances, but will look almost (asymptotically) free when probed at large Q 's.
- This property manifests in deep inelastic scattering, in which scattering of high Q photons against nucleons may be interpreted in terms of quasi-free partons.
- The strong coupling varies appreciably with Q , an effect that can be measured experimentally



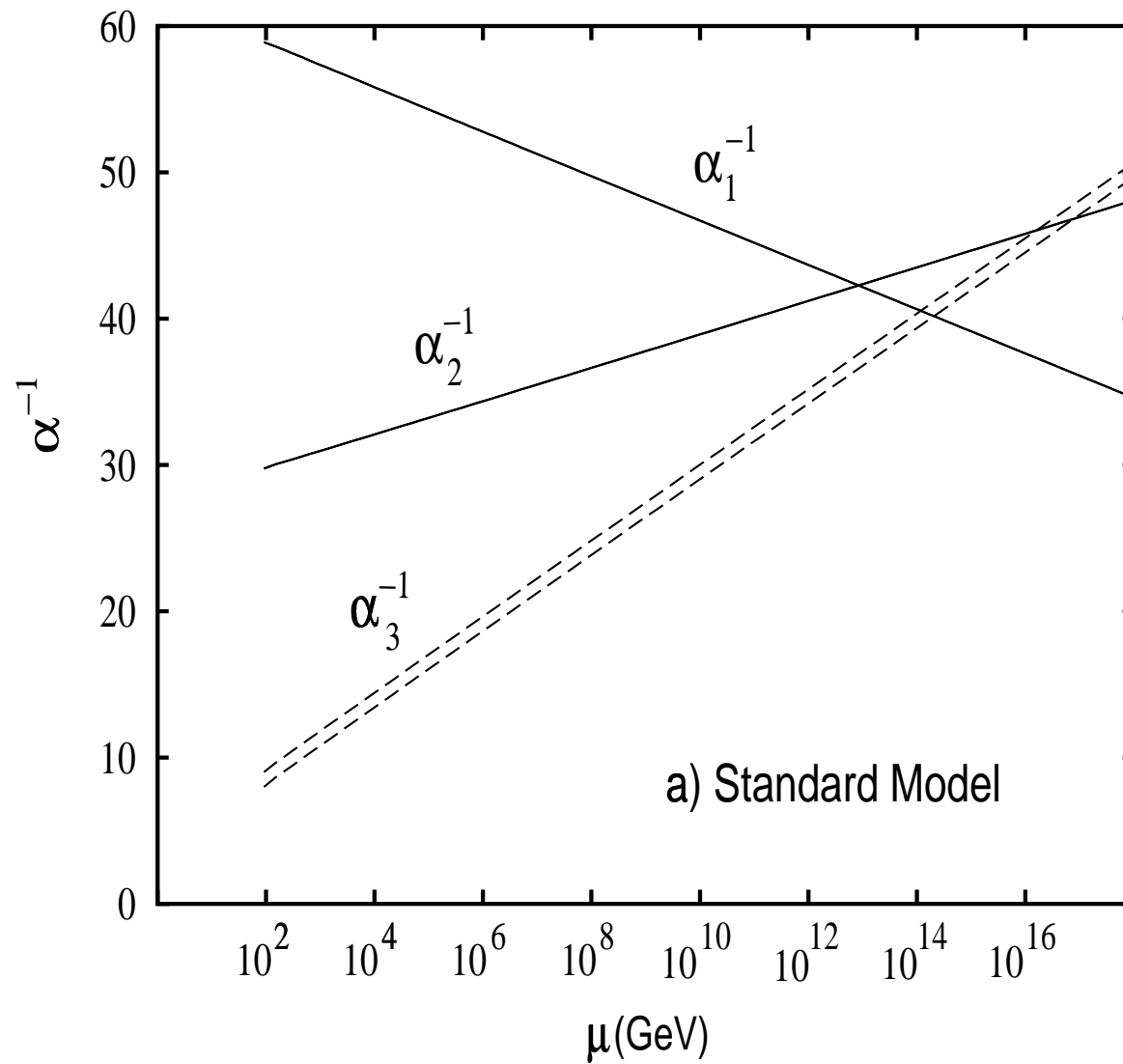
Fundamental couplings

- From a modern perspective, the Standard Model is assumed to be only an effective theory.
- It provides a good description only up to a maximal energy Λ
- To describe physics at energies beyond Λ , a new theory is necessary. Couplings at shorter distances are less affected by vacuum polarization corrections and are associated with more fundamental theory.
- By studying the evolution with Q of a coupling, we can make contact between the coupling we observe and the more fundamental one, determined by the UV completion of the present theory.

Unification of Couplings

- How can we probe if all low interactions possess a single fundamental description at short distances ?
- A hint would be present if, at some scale M_G , all couplings $\alpha_i(M_G)$ acquire equal values
- In the Standard Model, the weaker the coupling, the smaller the value of its corresponding b coefficient
- Couplings converge at short distances !
- Unification, however, is far from perfect, suggesting that the picture is not complete.

Running Couplings in the Standard Model



Testing Coupling Unification

- The value of gauge couplings evolve with scale according to the corresponding RG equations:

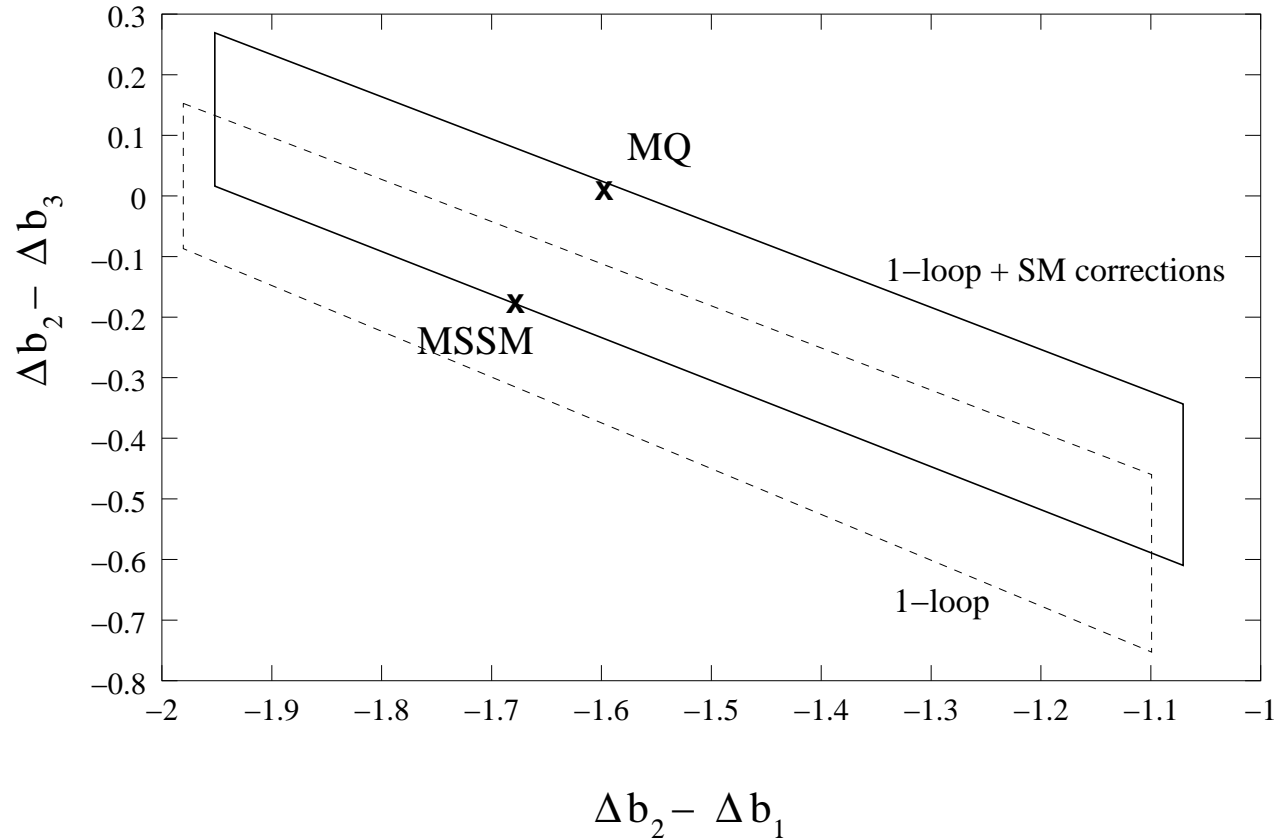
$$\frac{1}{\alpha_i(Q)} = \frac{b_i}{2\pi} \ln \left(\frac{Q}{M_Z} \right) + \frac{1}{\alpha_i(M_Z)} \quad (7)$$

- Unification of gauge couplings would occur if there is a given scale at which couplings converge.

$$\frac{1}{\alpha_3(M_Z)} = \frac{b_3 - b_2}{b_2 - b_1} \frac{1}{\alpha_1(M_Z)} - \frac{b_3 - b_1}{b_2 - b_1} \frac{1}{\alpha_2(M_Z)} \quad (8)$$

- This leads to a relation between $\alpha_3(M_Z)$, $\alpha_{\text{em}}(M_Z)$ and $\sin^2 \theta_W(M_Z)$. In the SM, for the measured values of $\sin^2 \theta_W(M_Z) \approx 0.2315$ and $\alpha_{\text{em}}^{-1}(M_Z) \approx 127.9$ one gets a value of $\alpha_3(M_Z) \simeq 0.180$.
- This is very different from the experimental value $\alpha_3(M_Z) \approx 0.120$.
What is needed to make it work ?

Necessary change of $\Delta b_i = b_i - b_i^{SM}$ to obtain Unification



These values should be compared with the contribution of a chiral fermion in the fundamental representation of $SU(N)$, $\Delta b_f = -1/3$.

From low energies to high energies

- Adding particle representations in order to fulfill the above conditions is a possible way to go
- A more sensible attitude would be to look for well motivated theories and check if Unification of Couplings works
- I shall present a few examples of such theories :
Supersymmetry, Warped Extra Dimensions and Beautiful Mirrors
- Examples motivated for different reasons, but they all lead to coupling unification

Supersymmetry and the Origin of Mass

In quantum field theory, a fermion mass term can be written as $\bar{\psi}_L \psi_R + h.c.$.

- Since left- and right-handed fermions transform differently under the gauge group, explicit fermion mass terms are forbidden in the SM.
- The only mass parameter is m_H^2 ,

$$V = m_H^2 H^\dagger H + \lambda (H^\dagger H)^4 \quad (9)$$

- If $m_H^2 < 0$ minimum at $H = (0, v)$ and the gauge symmetry is spontaneously broken.
- Fermions and gauge bosons acquire masses proportional to v .

$$\mathcal{L}_{mass} = -h_t \epsilon_{ij} \bar{Q}_i H_j t_R - h_b H^\dagger \bar{Q} b_R, \quad i = 1, 2 \quad (10)$$

- Masses protected by chiral gauge symmetry.

Stability of the relation $v \ll M_{Pl}$

- All masses controlled by m_H^2 .

$$v^2 \simeq -\frac{m_H^2}{\lambda} \quad (11)$$

- The symmetries of the Lagrangian are not enhanced for $m_H^2 \rightarrow 0$.
- Even if we assume $|m_H^2| \ll M_{Pl}^2$ at tree-level, it receives quantum loop-corrections of the order of M_{Pl}^2 .
- Natural value of v is M_{Pl} .
- The hierarchy of scales not understood in the SM
- Is there any symmetry that protects v from being affected by large quantum corrections ?
- Supersymmetry is such a symmetry

Properties of Supersymmetry

- Supersymmetry is a symmetry relating fermion and boson degrees of freedom.
- The supersymmetry generators are fermion operators, Q_α , $\alpha = 1, 2$.
- The supersymmetry algebra includes the space-time translation operator P_μ . In particular, for only one set of generators (N=1)

$$H = \frac{1}{4} (Q_1^\dagger Q_1 + Q_2^\dagger Q_2 + Q_1 Q_1^\dagger + Q_2 Q_2^\dagger) \quad (12)$$

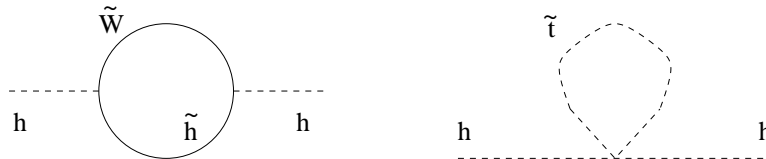
- If supersymmetry is preserved, $Q, Q^\dagger |0\rangle = 0$, $E_{vac} = 0$
- Local Supersymmetry implies Supergravity.
- Supersymmetry provides the ingredients for a Grand Unified description of all interactions, including gravity.

Loop Corrections to the Higgs Mass parameter



- Corrections depend quadratically on the cutoff,

$$\delta m_H^2 \simeq \frac{1}{16\pi^2} \sum_i (n_{B_i} g_{B_i}^2 - n_{f_i} g_{f_i}^2) \Lambda^2$$



- In supersymmetric theories the couplings and number of degrees of freedom of fermions and bosons are the same.
- No quadratic divergencies. Corrections proportional to supersymmetry breaking scale, $\delta m_H^2 \simeq M_{SUSY}^2 * \log(\Lambda/m_H)/16\pi^2$
- Natural hierarchy $\rightarrow M_{SUSY} \simeq 1 \text{ TeV}$.

Minimal Supersymmetric Standard Model

| SM particle | SUSY partner | G_{SM} |
|------------------|------------------------------|----------------------|
| (S = 1/2) | (S = 0) | |
| $Q = (t, b)_L$ | $(\tilde{t}, \tilde{b})_L$ | (3,2,1/6) |
| $L = (\nu, l)_L$ | $(\tilde{\nu}, \tilde{l})_L$ | (1,2,-1/2) |
| $U = (t^C)_L$ | \tilde{t}_R^* | ($\bar{3}$,1,-2/3) |
| $D = (b^C)_L$ | \tilde{b}_R^* | ($\bar{3}$,1,1/3) |
| $E = (l^C)_L$ | \tilde{l}_R^* | (1,1,1) |
| (S = 1) | (S = 1/2) | |
| B_μ | \tilde{B} | (1,1,0) |
| W_μ | \tilde{W} | (1,3,0) |
| g_μ | \tilde{g} | (8,1,0) |
| (S = 0) | (S = 1/2) | |
| \hat{H}_1 | $\tilde{\hat{H}}_1$ | (1,2,-1/2) |
| H_2 | \tilde{H}_2 | (1,2,1/2) |

Experimental Signatures of Supersymmetry

- R-Parity: $R = (-1)^{3B+L+2S}$. Supersymmetric Particles odd under R .
- If R-Parity Conserved:
Lightest Supersymmetric Particle (LSP) Stable
- Conservatively, LSP neutral.
- All supersymmetric particles decay into LSP
- $\tilde{q} \rightarrow q\chi_1^0$; $\tilde{l} \rightarrow l\chi_1^0$
 $\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 Z$; $\tilde{\chi}^\pm \rightarrow \tilde{\chi}_1^0 W^\pm$
- Missing Energy experimental signature of Supersymmetry !
- Most important, massive, neutral, weakly interacting particle is a good candidate for Cold Dark Matter, which is believed to contribute to 30 % of the total Universe Energy !

Unification of Couplings in the MSSM

- In the MSSM, predictions from unification of couplings are in good agreement with data.

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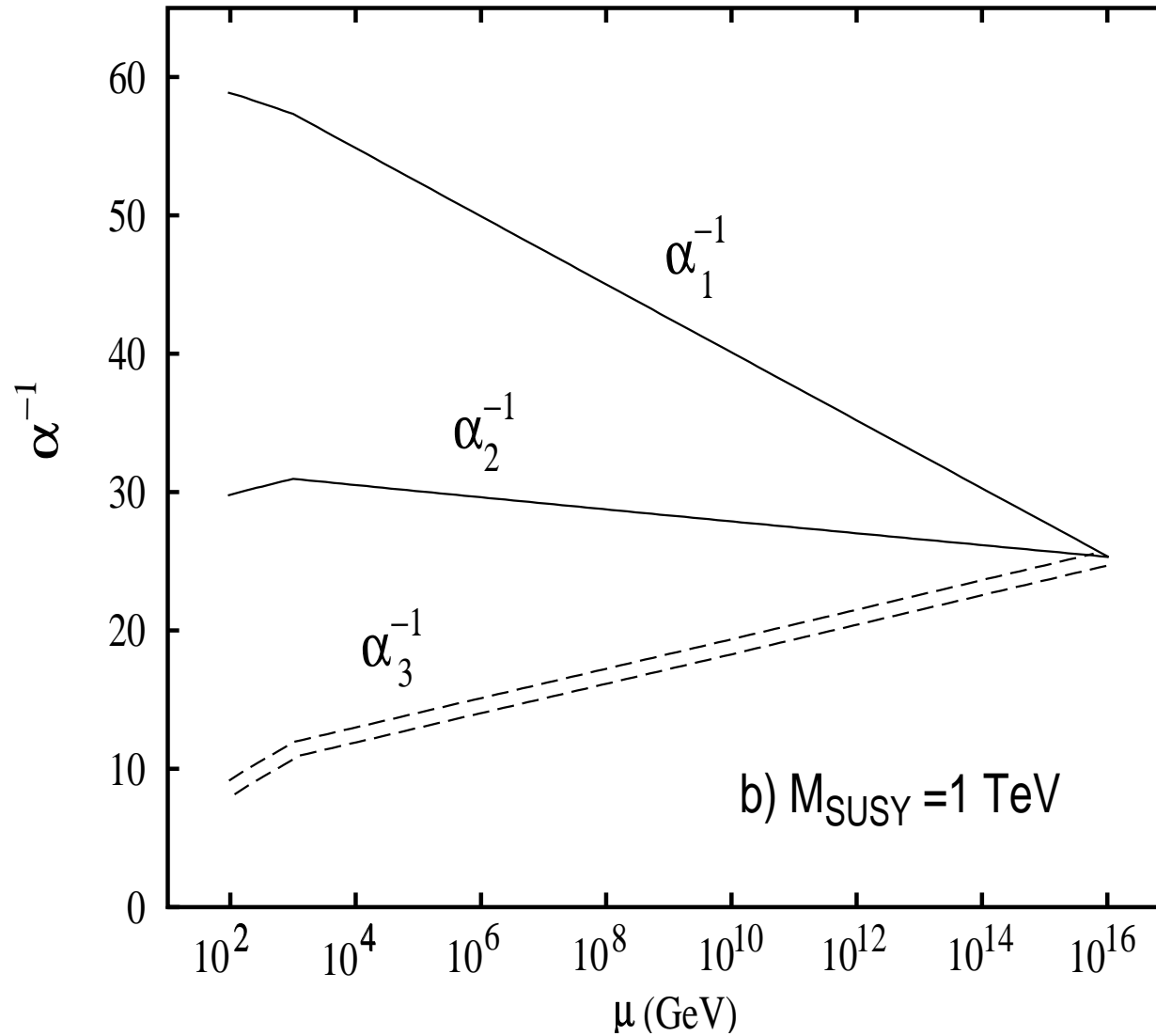
$$\alpha_3(M_Z) \simeq 0.128 + 4 (0.2315 - \sin^2 \theta_W) + \Delta_T(\alpha_3)$$

while experimentally $\alpha_3 \simeq 0.119 \pm 0.003$ and $\sin^2 \theta_W = 0.2315 \pm 0.0003$.

- The unification scale is $M_{\text{GUT}} = 10^{16}$ GeV, of about the Planck scale size !
- Low energy Threshold Corrections

$$\Delta_T \left(\frac{1}{\alpha_3(M_Z)} \right) = \frac{19}{28\pi} \log \left(\frac{T_{SUSY}}{M_Z} \right), \quad T_{SUSY} \simeq m_{\tilde{H}} \left(\frac{m_{\tilde{W}}}{m_{\tilde{g}}} \right)^{3/2} \quad (13)$$

Unification of Couplings in the MSSM



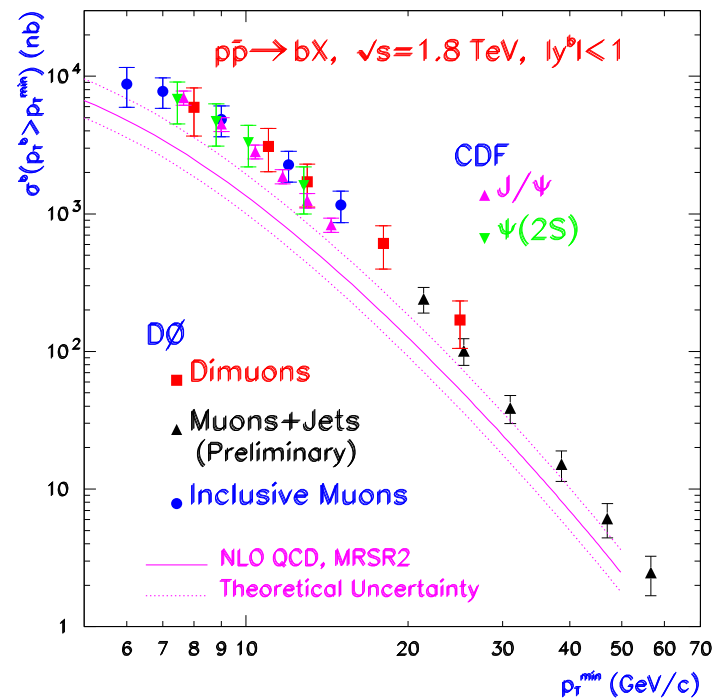
Unification of couplings is achieved with extremely good precision for a supersymmetry threshold scale $T_{\text{SUSY}} = 1 \text{ TeV}$

Precise Comparisons

- Well known models of supersymmetry breaking tend to induce $m_{\tilde{g}} \simeq 3.5 m_{\tilde{w}}$, and Higgsino masses of the order of a few hundred GeV.
- The scale T_{SUSY} is then naturally smaller than M_Z , leading to predicted values of $\alpha_3(M_Z) > 0.13$
- These values of $\alpha_3(M_Z)$ are larger than the experimental value, but can be moved down by assuming large threshold corrections at the GUT scale
- These large threshold corrections are not uncommon in realistic GUT models
- Alternatively, gluinos lighter than the ones arising in typical SUSY breaking models would be required
- Is there any hint in low energy data that light gluinos may be present ?

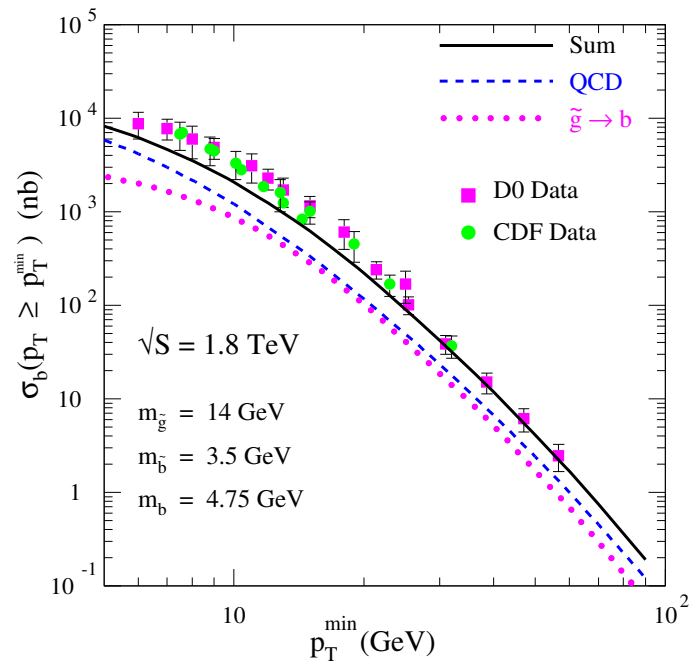
Bottom Quark Cross Section

- The bottom production cross section at the Tevatron ($p\bar{p}$ collider) exceeds NLO QCD by roughly a factor of **two**
- This discrepancy also appears in the CERN UA1 data



Supersymmetric Contribution

- A pair of light **gluinos** are produced which decay promptly to b and \tilde{b}



- The extra contribution is peaked about $p_T \sim m_{\tilde{g}} \sim 15 \text{ GeV}$ - right where the data shows the largest deviation!

Additional Predictions

- Relatively light gluino not excluded by other experiments. On the contrary, it is preferred by UA1 as well as by the Tevatron data.
- Light sbottom escapes constraints from Z-peak observables and precision electroweak observables, if $\sin^2 \theta_{\tilde{b}} \simeq 1/6 \pm 0.1$.
- Light stop, with mass of about the weak scale is generally predicted.
- The lightest CP-even Higgs decays mainly into bottom quarks. Difficult to observe at the Tevatron and LHC. Lepton collider would be necessary to detect this Higgs.
- B physics: Excess in equal sign bottom pairs predicted. Consistent with Tevatron Run I data. Critical test at Run II.

Prediction and Confirmation

- Production of equal sign pairs can be tested by looking at amount of B^0 and \bar{B}^0 modes produced.
- Neutral B^0 states oscillate into \bar{B}^0 states. Therefore, even in the SM, one measures a fraction of $B - B$ and $\bar{B} - \bar{B}$ pairs.
- Fraction of equal sign pairs measured characterized by a parameter $\bar{\chi}$, determined precisely at lepton colliders

$$\bar{\chi} = 0.118 \pm 0.007, \quad \frac{ES}{OS} = 2 \bar{\chi} (1 - \bar{\chi}) \quad (14)$$

- The above model provides an excess of equal sign bottoms (gluinos) at the production point induced by gluons. Excess not visible at lepton colliders
- A value of $\bar{\chi} = 0.15\text{--}0.18$ was predicted, depending on the light gluino mass, a range that is almost five standard deviations above the leptonic number. The recent value measured at the CDF Tevatron experiment is

$$\bar{\chi} = 0.152 \pm 0.007 \quad (15)$$

- Evidence of light gluinos ? (Unlikely, but very intriguing...)

Hints from Precision Electroweak Data

- Very good agreement between the Standard Model predictions and the measured value of electroweak observables, for a Higgs mass below 200 GeV.
- Oblique corrections to precision observables are logarithmically dependent on the Higgs mass.
- Fit to the data is improved for a Higgs mass of order 90 GeV, what suggest a Higgs with mass somewhat above the present direct limit, $m_H > 114$ GeV.
- Before reaching this conclusion, however, a critical analysis of the relevant observables used for the Higgs mass fit should be performed.
- Due to the accuracy in their measurements, hadron and lepton forward backward asymmetries measured at LEP play a very relevant role in the Higgs mass determination.

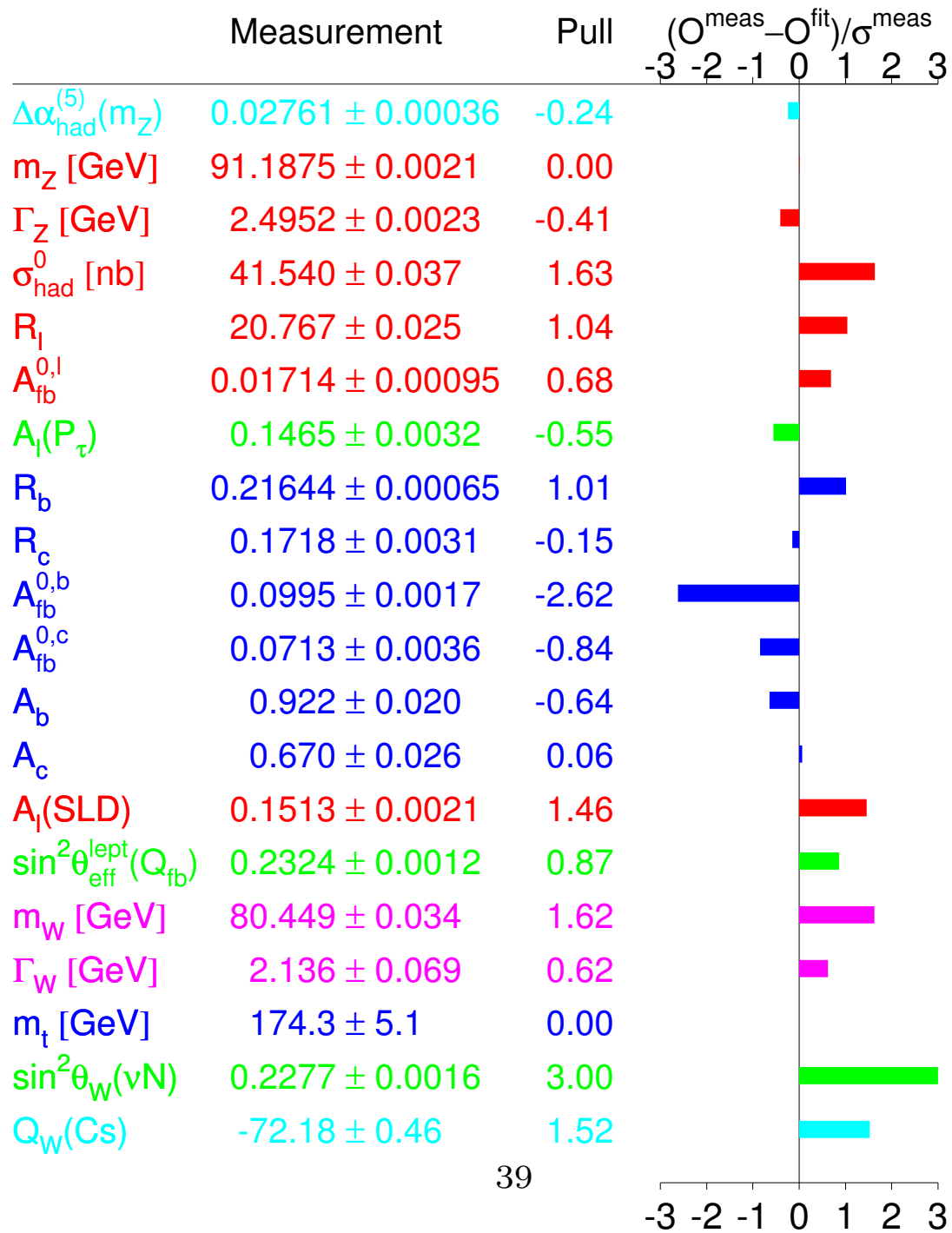
Testing the Standard Model

- There are simple ways of testing the Standard Model. One way is to check the width of the Z^0 -boson resonance in e^+e^- collisions. This is proportional to the sum of the squares of the Z^0 -fermion couplings.
- One can also test, for instance, the difference between the amount of quarks (or leptons) emitted in the direction of the e^- beam compared to those ones emitted in the opposite direction.
- It turns out that this is proportional to

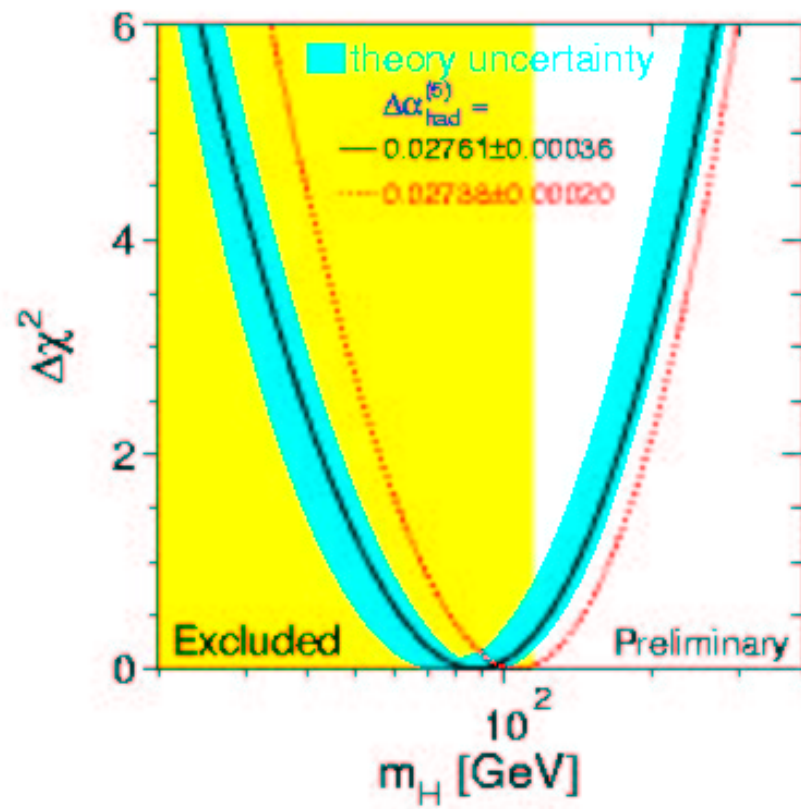
$$A_f = \frac{g_L^2 - g_R^2}{g_L^2 + g_R^2} \quad g_f = T_3 - Q \sin^2 \theta_W \quad (16)$$

where $g_{L,R}$ are the left- and right-handed fermion couplings to the Z^0 . Since T_3 and Q are well known, one can determine from here the value of $\sin^2 \theta_W$ rather precisely.

- The value of $\sin^2 \theta_W$ is governed by the ratio of the W and Z masses and quantum corrections that depend on the Higgs Mass
- Therefore, one can determine the unobserved Higgs mass quite precisely by using lepton and quark asymmetries.



Precision EW Data: Higgs Mass Fit



Higgs Mass Fit: The Problem

The value of $\sin^2 \theta_W$ coming from hadronic asymmetries,

$$\sin^2 \theta_W^{\text{eff}} \Big|_{\text{hadronic}} = 0.2324 \pm 0.00029$$

does not agree with the one coming from the leptonic ones

$$\sin^2 \theta_W^{\text{eff}} \Big|_{\text{leptonic}} = 0.23114 \pm 0.0002$$

- The EW-fit value of m_H is already below the direct lower bound. If we consider only the leptonic value, we would obtain even lower values ($m_H \simeq 50$ GeV).
- If the leptonic asymmetries would be correct, most of the region allowed by direct searches would be excluded at the 90 % C.L.
- In order to get the best fit value, a delicate balance between these two measurements of $\sin^2 \theta_W$ must be in action
- Evidence for a light Higgs boson is weakened by this fact. Moreover, as we shall see, this may be regarded as evidence of new physics.
- ['Lose-lose for the SM !', M.S. Chanowitz, hep-ph/010402]

Possible Solution. Hadron Asymmetries Wrong

- The lepton and hadron asymmetries are determined very precisely at $e^+ e^-$ colliders.
- However, it is much simpler to identify an electron or a muon than a given flavor of quarks (charm or bottom)
- Bottom asymmetries may be subject to “charm contamination” and one can doubt of the experimental accuracy claimed in these measurements
- If hadron asymmetries wrong, new physics will then be needed to compensate the SM quantum corrections and allow larger values of the Higgs mass
- What kind of physics can do that ?
Supersymmetry and warped extra dimensions provide examples. In both, consistent gauge coupling unification can be achieved.

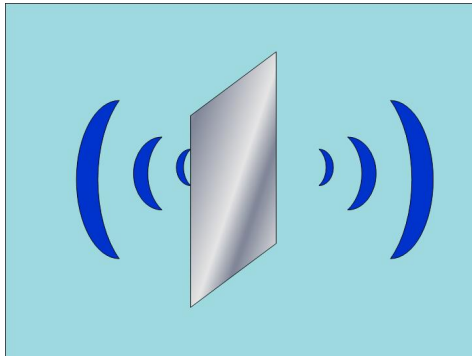
Effect of Supersymmetric Particles

- The physical effects needed to shift predictions to correct values can be achieved within the **MSSM**,
- Radiative corrections induced by supersymmetric particles tend to decouple quite fast with increasing masses. Large effects can only be obtained by light sparticles
- In this case, light sleptons are required to raise the Higgs mass value
 $m_{\tilde{\nu}} = 55\text{--}80 \text{ GeV}$,
 $\tilde{e}\text{'s} \gtrsim 95 \text{ GeV}$,
and light charginos as well.
- If charginos are light, particles observable at Tevatron collider.
Relevant signature: Trilepton channel

Effect of Extra Dimensions

Gravity in ED \Rightarrow fundamental scale, pushed down to electroweak scale by geometry

Metric: $ds^2 = e^{-2k|y|} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2 \quad \Rightarrow$ Solution to 5d Einstein eqs.



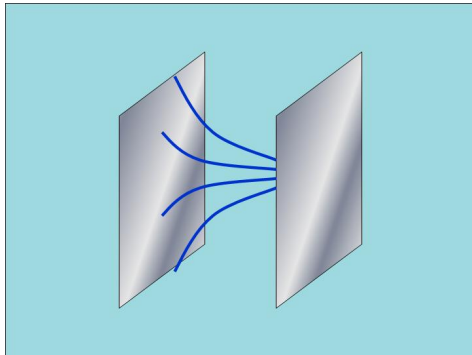
$k=0$ (flat)

gravity flux in ED \Rightarrow Newton's law modified:

$$M_{Pl}^2 = (M_{Pl}^{\text{fund.}})^{2+d} R^d$$

this lowers the fundamental Planck scale, depending on the size & number of ED.

$$M_{Pl}^{\text{fund.}} \simeq 1 \text{ TeV} \Rightarrow R = 1 \text{ mm}, 10^{-12} \text{ cm if } d = 2, 6$$



$k \neq 0$ (warped ED)

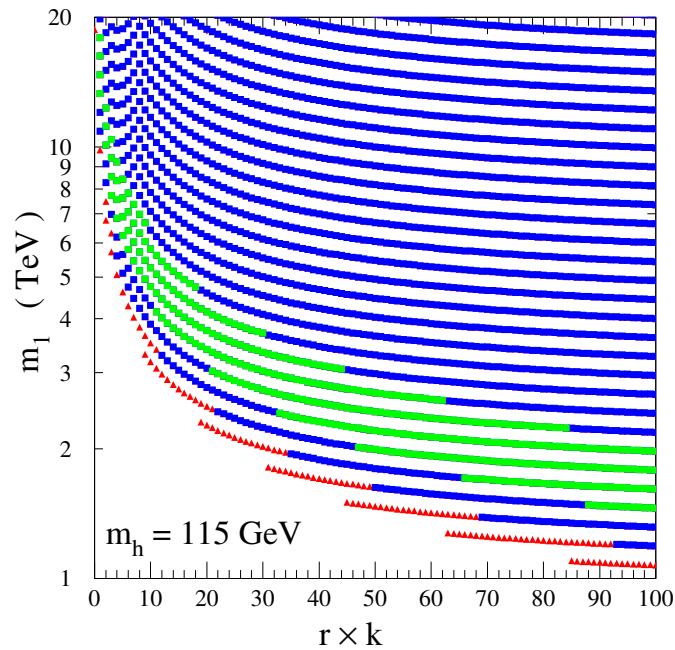
$$M_{Pl}^2 = \frac{(M_{Pl}^{\text{fund.}})^3}{2k} (1 - e^{-2kL})$$

fundamental scales: $M_{Pl} \sim M_{Pl}^{\text{fund.}} \sim v \sim k$

\Rightarrow Physical Higgs v.e.v. suppressed by e^{-kL}

$\Rightarrow \tilde{v} = v e^{-kL} \simeq m_Z$ if $kL \approx 34$

Effect of Warped Extra Dimensions



Carena, Ponton, Tait, C.W. '02

- Fermions and Higgs field localized on the IR brane
- Bulk gauge fields, with local brane gauge coupling $g_{IR}^2 = g_5^2/r_{IR}$
- If masses of KK modes below 3 TeV they may be observable at LHC.

Alternative Solution: Modification of $g_{L,R}^b$

- The idea is to modify the couplings of the bottom quark to the Z^0 gauge boson
- The necessary modification of the right-handed coupling is of order of 25 percent. Quantum-loop corrections too small
- Large tree-level corrections may be obtained via the mixing with a b -like quark
- Simplest case. Cancellation of anomalies and improvement of the fit to the data require the introduction of one doublet and one singlet mirror (vector) quark

$$\Psi_{L,R}^T = (\chi, \omega) \equiv (3, 2, 1/6)$$

and

$$\xi'_{R,L} \equiv (3, 1, -1/3)$$

- Since the mirror quarks have the same quantum numbers as the standard quarks, we shall call them Standard Mirrors.

Mixing Effects

Suppose there exists a charge $-1/3$ quark, b' , that mixes with b but not (significantly) with d, s .

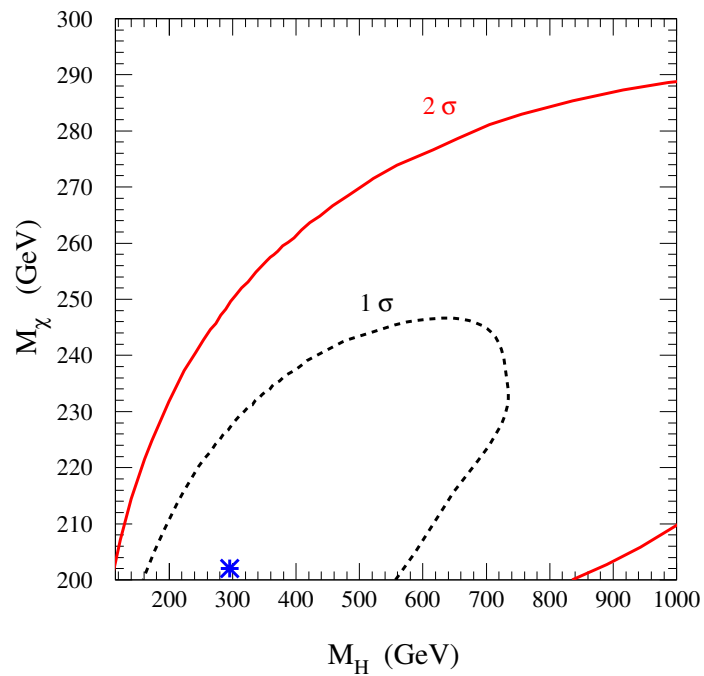
b' does not need to have same $SU(2) \otimes U(1)_Y$ quantum numbers as b .

$$J_\mu^3(b) = \frac{e}{s_W c_W} \sum_{ij} \bar{b}_i \gamma_\mu (L_{ij} P_L + R_{ij} P_R) b_j ,$$

$$L \equiv \begin{pmatrix} t_{3L} s_L^2 - \frac{1}{2} c_L^2 & -\left(t_{3L} + \frac{1}{2}\right) s_L c_L \\ -\left(t_{3L} + \frac{1}{2}\right) s_L c_L & t_{3L} c_L^2 - \frac{1}{2} s_L^2 \end{pmatrix} \quad R \equiv \begin{pmatrix} t_{3R} s_R^2 & -t_{3R} s_R c_R \\ -t_{3R} s_R c_R & t_{3R} c_R^2 \end{pmatrix}$$

$$\bullet \quad \delta g_L^b = \left(t_{3L} + \frac{1}{2}\right) s_L^2 , \quad \delta g_R^b = t_{3R} s_R^2$$

Standard Mirrors. The fit



Here M_1 denotes the mass of the top-like quark. The mass of the lightest b -like quark is $m_\omega = 1.25 M_1$, in order to get a good fit to the data

Unification of Couplings: Standard Mirrors

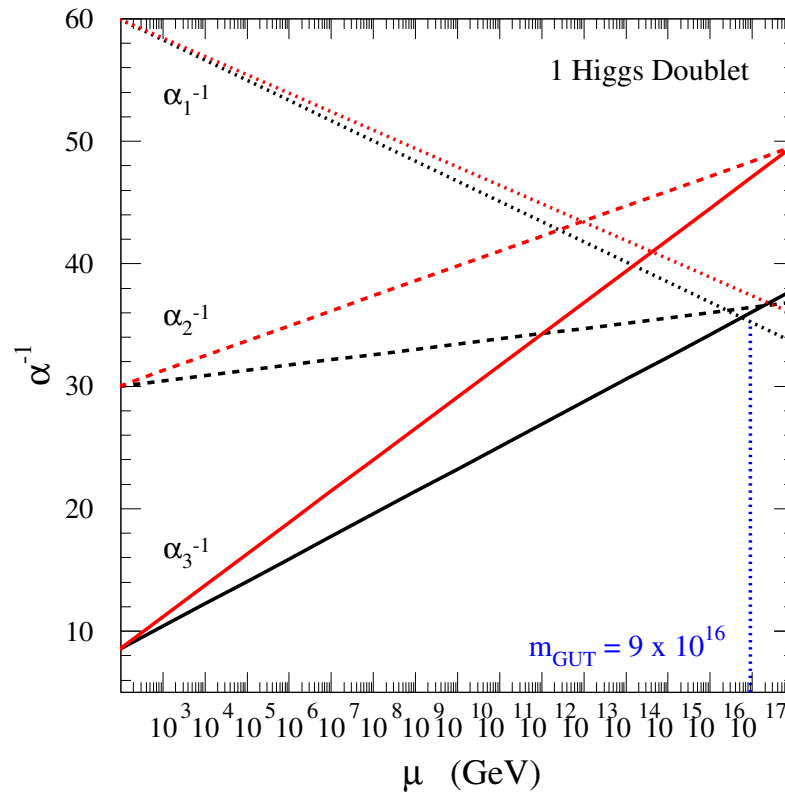
Standard Mirrors: We shall do a two-loop analysis. Will not take GUT threshold effects into account. One-loop beta-function coefficients:

$$\begin{aligned}b_3 &= 11 - \frac{4}{3}n_g - 2 \\b_2 &= \frac{22}{3} - \frac{4}{3}n_g - \frac{n_H}{6} - 2 \\b_1 &= -\frac{4}{3}n_g - \frac{n_H}{10} - \frac{2}{5}\end{aligned}$$

where n_g is number of generations and n_H is number of Higgs doublets.

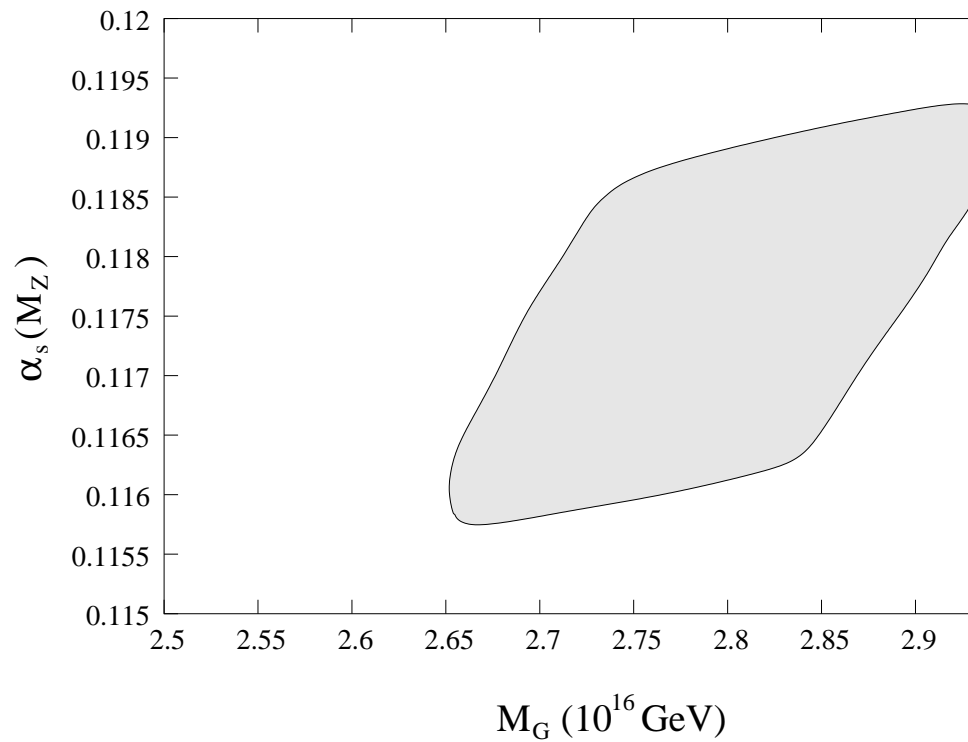
- Observe that the conditions for good unification are fulfilled

Unification in Standard Mirror Scenario with $n_H = 1$



Predicted values of $\alpha_3(M_Z)$ and M_G

Two loop predictions



- Large values of the unification scale
- Perfect agreement with the measured value of $\alpha_s(M_Z)$.

Higgs phenomenology

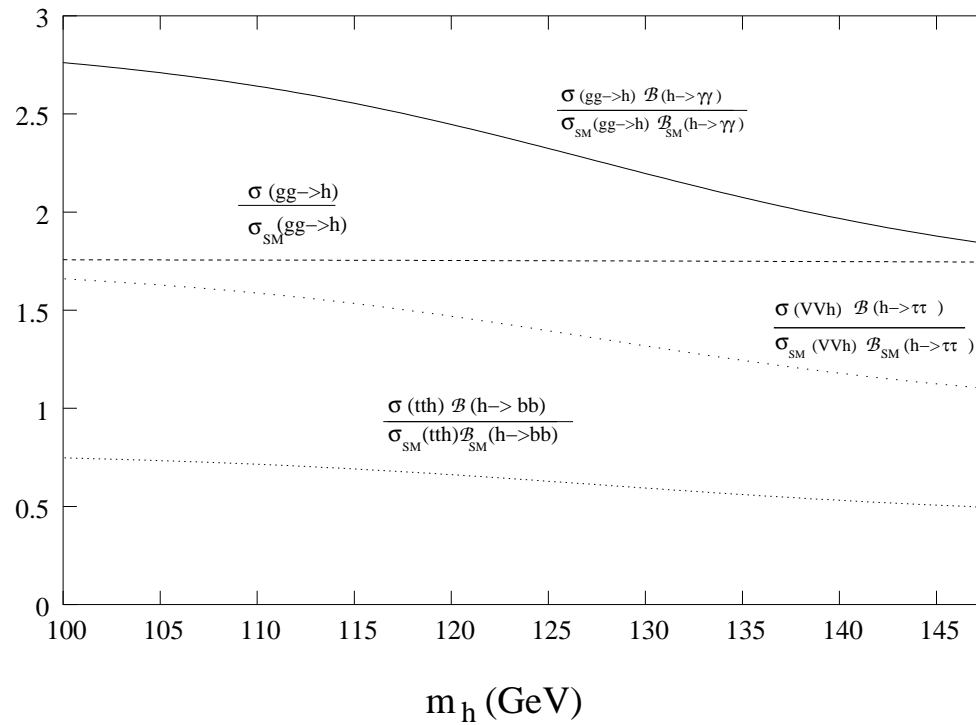
- In the Standard Mirror case, if $m_H < m_\omega + m_b$, Higgs will preserve the Standard decay channels, but with a modified b -coupling:

$$\frac{g_{Hb\bar{b}}}{g_{Hb\bar{b}}^{SM}} = \cos^2 \theta_R \quad (17)$$

Since $\tan \theta_R \simeq 0.7$, this leads to a reduction of order $2/3$ with respect to the SM coupling. For a Higgs heavier than $2m_W$, this will have only a mild impact on phenomenology.

- Second important effect: The presence of new quarks with relevant coupling to the Higgs increase the effective $H \rightarrow g g$ coupling.
- $H \rightarrow \gamma\gamma$ coupling only slightly modified.

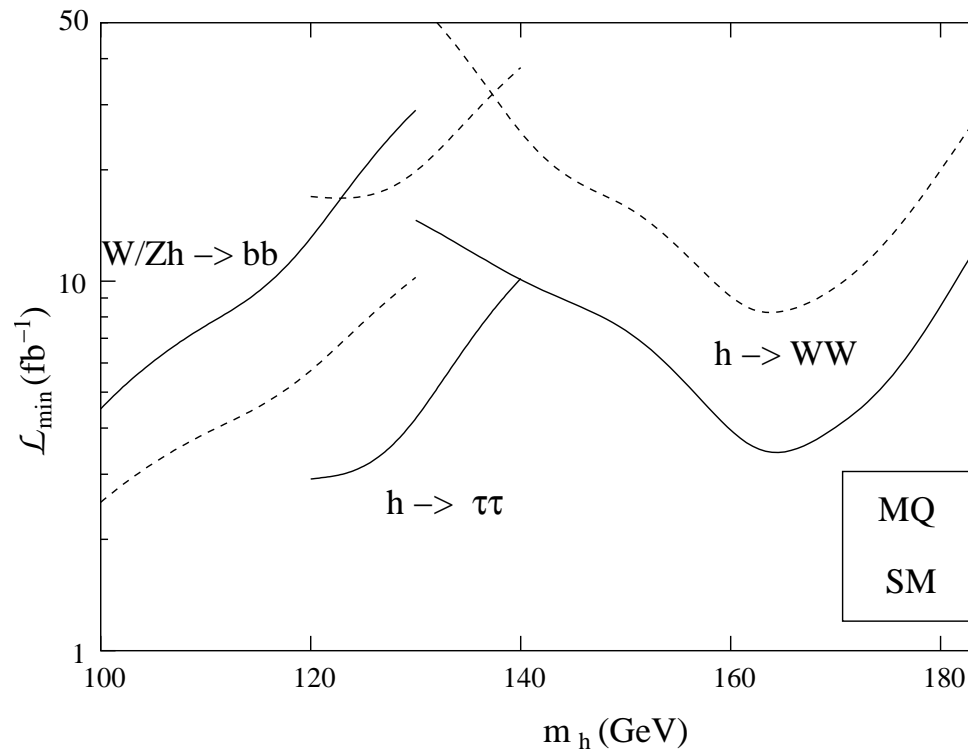
Relevant Higgs production rates: SM vs. Mirror Quark Model



Gluon fusion production, with $h \rightarrow \tau^+ \tau^-$ may be inferred from above ($\text{VV}h$ and $h\gamma\gamma$ couplings only slightly modified).

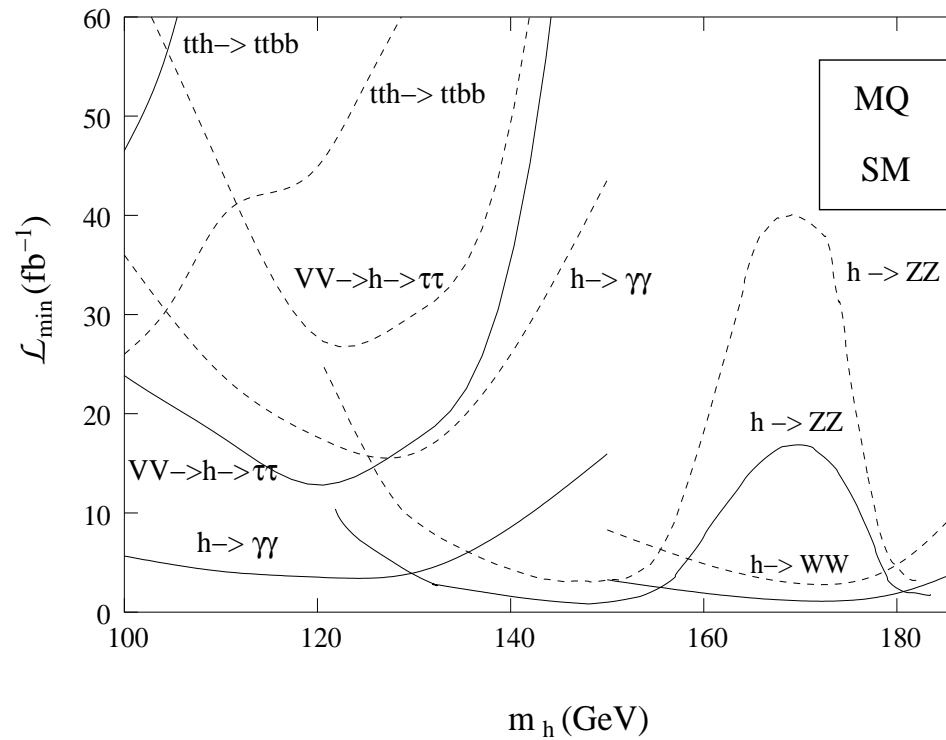
Higgs Searches at the Tevatron

Minimal luminosity for a 3- σ evidence of a Higgs.



- With 8 fb^{-1} of luminosity, a 3-sigma evidence is possible, up to Higgs masses of 180 GeV.
- The $\tau\tau$ channel is the most relevant one at low Higgs masses

Higgs Searches at the LHC



Significant improvement in all gluon fusion related channels.

Proton Decay

- If quark and leptons belong to common representations of a larger gauge group, some of the (heavy) gauge bosons mediate transitions between leptons and quarks
- An immediate consequence is proton decay.
- In supersymmetric theories, there are further sources of baryon number violation, associated with the supersymmetric partners of heavy Higgs bosons (dimension five operators)
- Precise rate strongly depends on the model and in the precise source of Yukawa couplings in the theory.
- Observation of proton decay would be a very powerful hint of a unified theory of particle interactions

Proton Decay in minimal theories

- In minimal supersymmetric SU(5), dominant decay mode is provided by

$$P \rightarrow \bar{\nu} K^+ \quad (18)$$

Proton lifetime tends to be at the edge the present bounds, and therefore strong constraints in this model are obtained

- Larger gauge groups allow more freedom, but the absence of proton decay remains a strong constraint in these models
- Within supersymmetry, constraints can be relaxed by going to flipped SU(5) ($SU(5) \times U(1)$), since dimension five operators are absent.
- In the non-supersymmetric model discussed above, there are no dimension five operators induced even within standard unified theories.
- Proton stability, then, improves dramatically in this model (as well as in flipped SU(5))

$$\tau(p \rightarrow \pi^0 e^+) = 3 \times 10^{36 \pm 1} \text{ years} \quad (19)$$

well in excess of the Super-Kamiokande bound on $\tau(p \rightarrow \pi^0 e^+) = 5.3 \times 10^{33}$.

Conclusions

- Much progress has been done in our pursuit of a Unified Theory of Particle Interactions
- Neutrino, as well as the quark and lepton masses and mixings provide additional avenues to explore the nature of the fundamental theory
- Low Energy Data give hints of the existence of such a theory, but specific implementation still uncertain
- Most likely low energy manifestation: MSSM
- Unification with Gravity requires to go beyond field theory.
- String Theory most promising candidate, but multiplicity of (mostly unrealistic) vacua makes very difficult the connection with experiment
- Guidance from experiment is needed
- The Tevatron, the LHC, a high-energy lepton collider and the Cosmos will provide us the additional relevant information to know if we are in the right path.